

# Anton Bakker

## MoMath Virtual Exhibit

ALTERNATIVE PERSPECTIVES



**The National Museum of Mathematics**  
*presents:*

# **ALTERNATIVE PERSPECTIVE**

The Art of Anton Bakker

Curated with texts by  
Doris Schattschneider and Tom Verhoeff



Virtual access at **[composite.momath.org](https://composite.momath.org)**

Opening Saturday, August 15, 2020

## Beauty, creativity, exploration...

The ability to enhance perceptions and change perspectives...

The simple elegance—or the delicious complexity—of pattern, texture, and color...

Art, or math?

There's an artistry to mathematics that captivates those who peer closely—and mathematical interpretations of art that can illuminate and elucidate.

Welcome to *Composite*, the gallery at MoMath, where art and math intersect to provide a unique perspective: a chance to perceive the world from a different angle, and to come away with a new, often surprising, understanding.

The National Museum of Mathematics is delighted to launch a groundbreaking virtual exhibit in *Composite*, the gallery at MoMath, featuring the elegant and beautiful mathematical sculpture of Anton Bakker. Combining a visually stunning display with unprecedented three-dimensional interactivity, *Alternative Perspective* takes the visitor on a journey of surprise and wonder with an added twist—a change in perspective seems to change the very reality of the object before you. Bakker's unique method of creation and exploration of mathematical structure and symmetry results in a collection that bridges two worlds, expressing both the mathematical nature of art and the artful nature of mathematics. We hope you will join us in enjoying, engaging, and discovering.



Cindy Lawrence  
CEO and Executive Director  
National Museum of Mathematics

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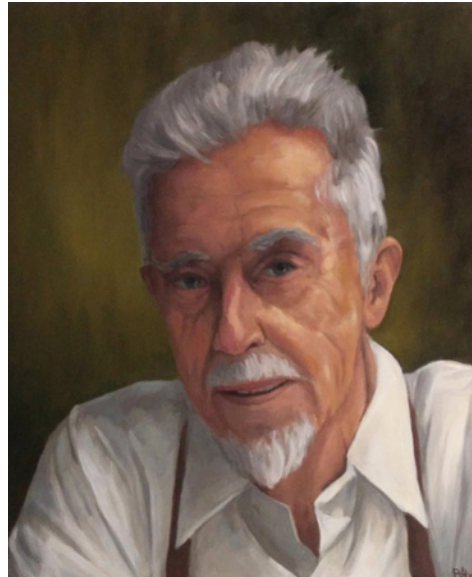
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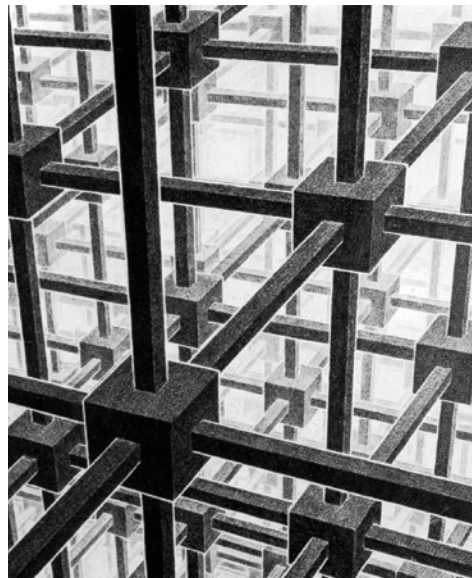
Koos Verhoeff by Péta Vlieger



M.C. Escher by Péta Vlieger



Fishes in Waves by M.C. Escher,  
signed and gifted to Koos  
by M.C. Escher



Cubic Space Division  
by M.C. Escher

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## ABOUT THE ARTIST

Anton Bakker is a contemporary artist specializing in sculpture and its digital possibilities. He has been influenced by the people and experiences of his life in the Netherlands, France, and in the United States, where his artistic practice has been based for more than 30 years.

While growing up in the Netherlands, Bakker met mathematician and artist Dr. Jacobus “Koos” Verhoeff at the suggestion of his sister’s classmate. What began as a simple introduction over a shared interest in computer technology turned into a 40-year artistic collaboration. Koos was a professional acquaintance and informal advisor on mathematical matters to the famed M.C. Escher. As an expression of his gratitude, Escher gifted Koos one of his prints. It was through Koos that Bakker became influenced by Escher’s unprecedented approach to perspective.



As their relationship developed, Koos and Bakker began to explore computer-based methods to find intriguing and beautiful paths within cubic lattice structures and polyhedra. Cubic lattices form the basis of the most stable molecular forms of many elements.

In the 1980s, Bakker moved to the United States, and he and Koos had their first joint sculpture exhibition in Albany, New York. Subsequently, Bakker leveraged his growing knowledge of computer science to pursue a career in technology, landing a position that required relocating to Paris for much of the 1990s. While in Paris, Bakker resumed regular face-to-face work sessions with Koos. Together, they created multiple lattice-derived sculptures that were exhibited throughout Europe.

Meanwhile, Bakker was at the forefront of a new tech field, working with innovators in Belgium to explore the possibilities of 3D printing. Upon returning to the U.S. in 1997, he started a business centered on data analysis all the while maintaining his artistic practice. His solutions for practical design and construction problems opened new possibilities for connecting lattice points with curved and polyline paths. By applying these techniques at both small and large scales in steel, bronze, and in virtual reality, Bakker has created unique sculptures that have been collected privately and publicly throughout the United States and Europe.

Bakker sold his tech business in 2018, shortly after the death of Koos, to devote himself to art full time. Today, he uses technology to compose paths in order to find those with a unique beauty that transforms as the viewer shifts their point of view. With the aid of a computer interface, Bakker searches vast lattice expanses to identify points that generate intriguing paths in a quest to challenge the limits of perception and perspective.



## ARTIST STATEMENT

As a sculptor creating digital and physical forms, I strive to take the viewer on a journey of truth-discovery by asking them to engage with various perspectives. Using custom-built technology, I create paths by connecting points in space. The curved and polyline paths that I compose are not arbitrary, rather, they are patterns derived from nature's archetypes.

The human attraction to symmetry extends deep into the unconscious realms of our minds. Natural patterns and symmetries also play a key role in present-day technology. For 40 years, I have used technology both in my artistic explorations with my mentor, Koos, and in my business to analyse patterns. I now use technology solely to discover the beauty that hides in the minuscule yet vast world of atomic lattices.

One way that I explore perspectives is by constructing objects at vastly different scales and in multiple dimensions. The viewer's relationship with my work changes whether they walk around a sculpture in a home, as part of an outdoor installation, or in a virtual landscape. My sculptures reveal dynamic symmetries that ask the viewer to reflect on the beauty and multiplicity of perspectives inherent in all things.

# SCULPTURE EVOLUTION



Cubic Lattice

## THE EVOLUTION OF AN ANTON BAKKER SCULPTURE

Anton Bakker's artistic realm is 3-dimensional space, punctuated by pinpoints of light in a "cubic lattice." To envision this, think of cubes all exactly the same size, stacked neatly in all directions, matching edges, faces, and corners, to fill space. Then light up the corners and remove all but these lights. This is the cubic lattice. The artist decides on a path of line segments (called the "generator") that connect some of these points, building in certain symmetries such as repetition, reflection, and reversal.

The instructions for marking out the generator are encoded in a special language that specifies how to travel from point to point, as in "turtle geometry." (Think of a robot moving in space, directed how to travel to connect certain points.) The coded information is fed into Bakker's computer program, and the program can search and find thousands of ways in which to repeat and connect copies of the generator to form non-intersecting simple loops and display images of them. The artist specifies how far these connected paths can venture from the initial point before they must follow a return route.

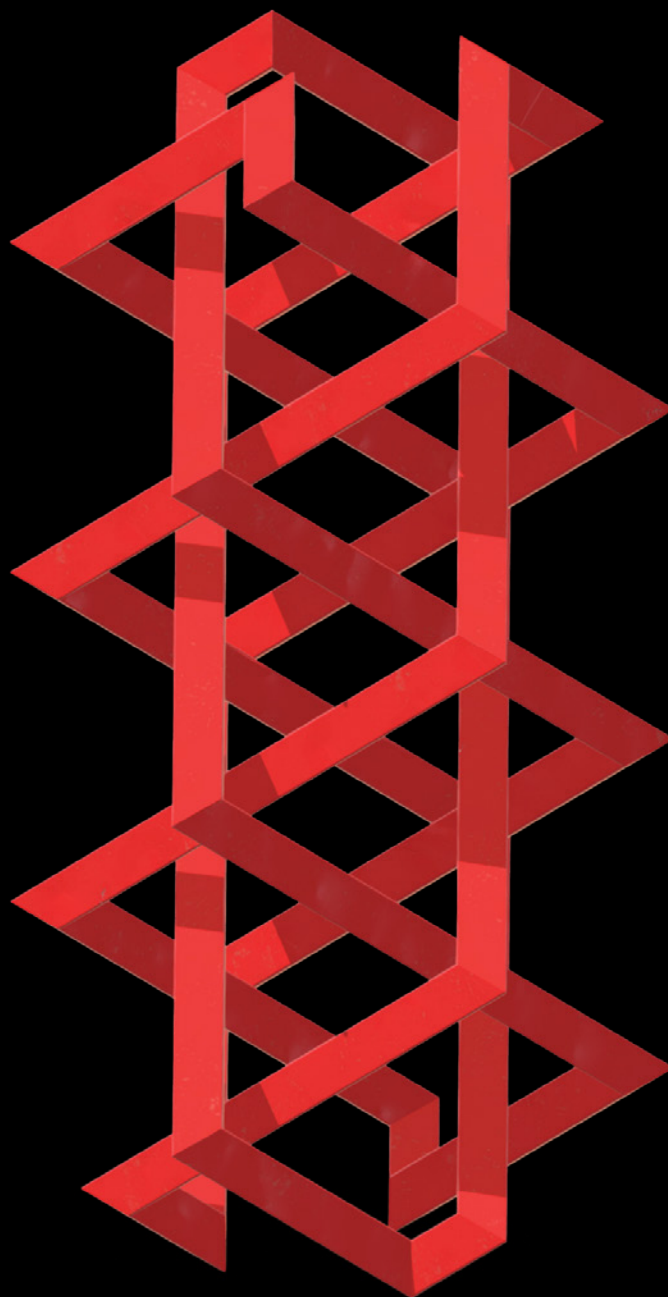
The artist can ask the program to filter the results, choosing or discarding those loops that are knotted, for example. He then chooses a few that might have aesthetic potential and gives life to these "stick figures" or "wire frame forms" by coating them. They can be coated uniformly, making all cross sections exactly the same — all circles, or triangles, or squares. But by smoothing the sharp corners of the stick figures and varying the thickness and width of the coating, the figures are transformed into sinuous, ever-flowing streams. The artist can view the results on his screen, turning the virtual figure through every angle to see its symmetries and the 2-dimensional illusions it creates. In the final stage, he decides the size and medium in which to have it fabricated as a tangible sculpture.

## POLYLINES

Connect the dots is a simple game we play with pencil on paper to outline a hidden image. We connect numbered dots, in order, with line segments. Most often, these connected edges form a non-intersecting loop that can be called a simple closed polyline path, a polygonal circuit, or just a polygon. The game can be played as well in 3-dimensional space, where the numbered dots can be chosen from an infinite regular array of points called a lattice. A familiar lattice is the cubic lattice, a 3-d version of square graph paper: its points determine the corners of neatly stacked cubes that fill space.

Instead of numbering dots to be connected, a polyline path can also be traced out (on the plane or in space) by giving a series of instructions for the pencil (or “turtle”) to follow. Move  $x$  units far in direction  $y$ , then Turn through angle  $z$ , then continue in a sequence of Moves and Turns for perhaps different  $x$ ,  $y$ , and  $z$  values. In space, a Turn may also include a roll, like an acrobatic airplane move. To complete a circuit, the last Move connects the path to the starting point. In the plane, instructions that repeat exactly the same Moves and Turns trace out regular polygons: equilateral triangle, square, hexagon, etc. In space, polyline paths can dip and twist in many directions before completing a circuit.





OPUS 951465

## CURVES

Polylines have abrupt, often sharp corners as they trace out a circuit. These paths do not flow, they jerk. To smooth a polyline path into a flowing curve, Bakker uses what mathematicians call spline interpolation. This is a bit like fitting a thin springy strip of steel around a set of pegs to form a curved path that touches each peg.

Cubic functions (the simplest is  $y = x^3$ ) have curvy, S-shaped graphs. They have the remarkable property that, given four points (not all on a line), there is a cubic function whose graph goes through those four points. If the four points are fairly close to each other, the piece of the cubic curve running through them (called a spline) closely approximates line segments that connect the points. Using splines, Bakker can replace each sharp V-shaped corner of a polyline path with a U-shaped curve. The result is a smoothly curvaceous circuit that travels through all the corners of the polyline path.

The curved loop that results from smoothing a polyline circuit in space is merely a skeleton doodle with no thickness and no body. This must be provided by the artist. A simple thickening coats the curve so it has a uniformly shaped cross-section such as a circle (which produces a tube covering), a square, or triangle. The width and thickness of the curve's covering can be varied for aesthetic reasons. This can suggest a change of speed and spread as the curve flows, much like water flowing in a creek that meanders through changing terrain.



OPUS 980011

## KNOTS

Knots are familiar shapes yet can be dauntingly mysterious (especially when trying to untangle a messy one). Knowledge of some knot formations is a necessity for sailors, yet there is much we don't know about knots – so mathematicians study “knot theory.”

Mathematical knots are closed. They do not have two loose ends, like shoelaces that can be untied. To make the simplest mathematical knot, take a length of string or flexible wire and bend it so the two ends cross each other. Now take the end that is “on top” and twist it to go under, then over the other end. Finally, glue the two ends together. This is called a trefoil knot. In its most symmetric presentation, it looks like three identical rings woven together. It is impossible to undo this or any mathematical knot without cutting it.

When Bakker gives instructions to his computer program to connect copies of a polyline generator in order to form closed circuits in space, some of the circuits among the thousands produced may be mathematical knots. The program contains a filter that can identify which of the circuits are knots, and from these the artist can select what becomes the basis for a knotted sculpture.



OPUS 325846

## MÖBIUS TWISTS

A thin strip — of paper, say, or springy metal or wood — can be bent into a ring by joining its two ends. If you don't twist the strip, you get a simple cylindrical ring, like the hoop that holds together the staves of a wooden barrel. But if the strip is twisted before the ends are joined, the ring that is formed has what is called a Möbius twist, named after the mid-19th century German mathematician August Ferdinand Möbius (although the form was known to the ancient Romans). The form has some surprising properties. A single twist of  $180^\circ$  will join the top edge of one end of the strip to the bottom edge of the other end, producing a one-sided loop. That is, you can trace a continuous path along the center line of the loop, parallel to the edges, until you return to the starting point, and in doing so, you will have traveled along the center line of both the front and back side of the original strip.

The polyline circuits and their curved counterparts that are the skeletons, for Bakker's sculptures often twist as they visit points in the cubic lattice. Möbius twists can become apparent when the skeletons are coated so their cross sections have rectangular shapes. The cross-sections travel like a roller coaster car on the skeleton path, sweeping through the sculpture's circuit. The cross-sections of the coating are varied for aesthetic interest, but also must vary so that at the beginning and end of the circuit the cross sections match and can fuse.



OPUS 247632

## FRACTALS

The term “fractal” suggests fracturing or splitting. Mathematicians use the term to describe a figure made up of an infinite number of parts, each part a scaled version of a single part, with the scaling constantly diminishing the size of repeated parts. Look closely at a fern, or a head of broccoli. These are finite, or partial, versions of fractals: when you look closely at the parts that compose them, the parts are smaller versions of the whole.

A fractal can be created by beginning with a particular figure or shape and then following a set of recursive instructions. That is, an action is performed on the shape such as adding to it, or splitting it, which creates new smaller shapes similar to the original. The instructions are then applied to these new smaller shapes, and the process repeats again and again, ad infinitum. A fractal tree, for instance, can be created by splitting the original “trunk” into two thick branches that are smaller copies of the trunk. These two branches in turn each split in exactly the same manner, and the process repeats again and again as smaller and smaller branches grow on the tree.





Koos Pythagorean Fractal Tree

## ILLUSION

Optical illusion is a playground for artists. Every artist who depicts a solid, “real” object on a 2-dimensional canvas strives to create the illusion that the object viewed is 3-dimensional and can employ a host of visual tricks to accomplish that goal. For centuries, masters of trompe l’oeil (fool the eye) have created illusory cornices, windows, balconies, and doors to enhance or enliven otherwise bare walls.

In 3-dimensional space, artistic tricks are not needed to produce illusory images. Instead, our eyes and brains themselves supply us with illusions: we see, or think we see, something that is not really what it is. If you look straight into a shallow round bowl, for example, your brain might not be able to determine if the shape is concave (scooped out) or convex, like a mushroom’s dome.

The viewpoint from which we observe a 3-dimensional object can be crucial to our “seeing” and understanding the object. Unless the object is transparent, we cannot see parts of it that are obscured by other parts covering them. We can only see a projection, or shadow, of what is directly in front of us. Bakker capitalizes on this property so that his sculptures provide teasing optical puzzles: if I observe the sculpture from this viewpoint, can I guess the full sculpture’s shape? Only by rotating the sculpture in space, viewing it from many angles, can you discover its surprising symmetries.



Ode to M.C. Escher  
OPUS 548001

## SPIRALS

In the plane, when an object rotates about a fixed point while simultaneously moving away from that point, it traces out a spiral path that constantly curves outward. Rope is often coiled in this manner on a boat's flat deck. A spiral path can also begin far away from a fixed point when an object rotates about that point while moving ever closer to that point. Think of the tightly wound head of an emerging fern.

In space, a spiral path is traced by an object that rotates about a fixed axis while moving away from that axis, and has the additional freedom to move upward, like a waterspout, or like the ridges of a screw traveling from tip to head. A spiral path can even begin at a point and rotate while moving outward and upward, and then, reaching the widest distance from its axis, spiral inward about the same axis while continuing its upward journey. This is the path of a theoretical ship that travels the globe from south pole to north pole with its compass always at a fixed angle to the globe's meridians; the path is called a loxodrome, or rhumb. In space, the artist has the freedom to create a spiral path about one axis, then have the curve turn to spiral about a different axis.



OPUS 185131

## STARS

Mathematical stars, unlike celestial ones, are pointy, usually symmetric shapes that burst outward from a central “core.” In a plane, regular star-shaped polygons are traced by line segments connecting equally spaced dots on a circle. If the dots are connected in cyclic order, a regular convex polygon results. But if the dots are connected in order, skipping over one dot each time, a “star polygon” will result. If the number of dots is odd, the traced path will return to the starting point, completing the star. But if the number of dots is even, the traced path will close after visiting only half of the points, and a second path must connect the remaining points to produce the star. The final star consists of two identical convex polygons, one turned to overlap the other, like two triangles composing a familiar 6-pointed star. Other star polygons can be traced in a similar way, by repeatedly skipping over more than one point as dots on a circle are connected.

In space, intricate polyline paths can trace out circuits with star-shaped projections. Here, with 3-dimensional freedom of movement, a path can even trace out a figure having all 90° corners, traveling over and under itself, at times in one plane, then in a plane perpendicular to that one, repeating the same travel instructions. The completed circuit might be an intricate knot where, in each of several projections, we see a symmetric star.



OPUS 475850





Perspective 1





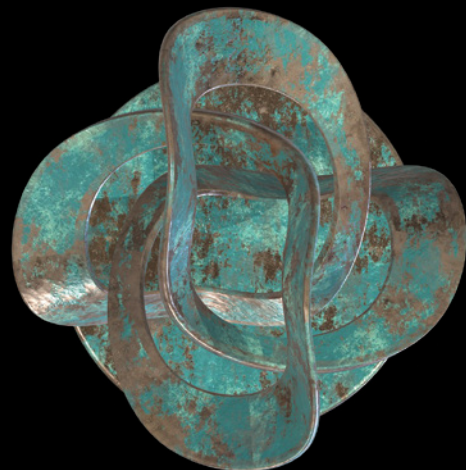
Perspective 2



Perspective 3



Perspective 4



Perspective 5



Perspective 1



Perspective 2



Perspective 3



Perspective 4



Perspective 5



Perspective 1



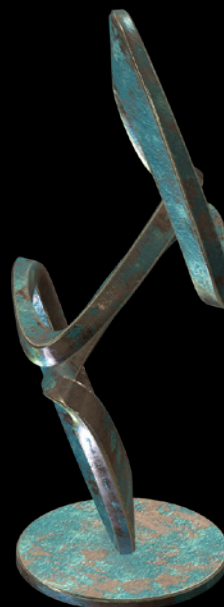
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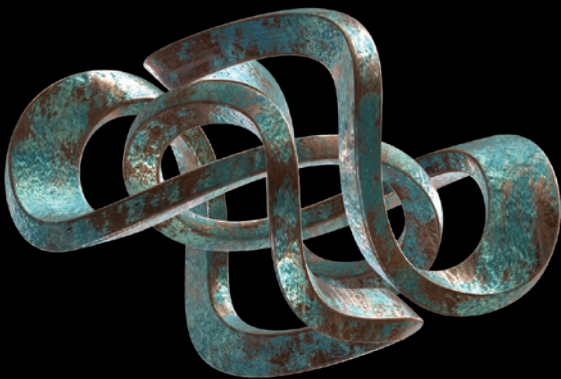
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Perspective 4



Perspective 5



Perspective 1





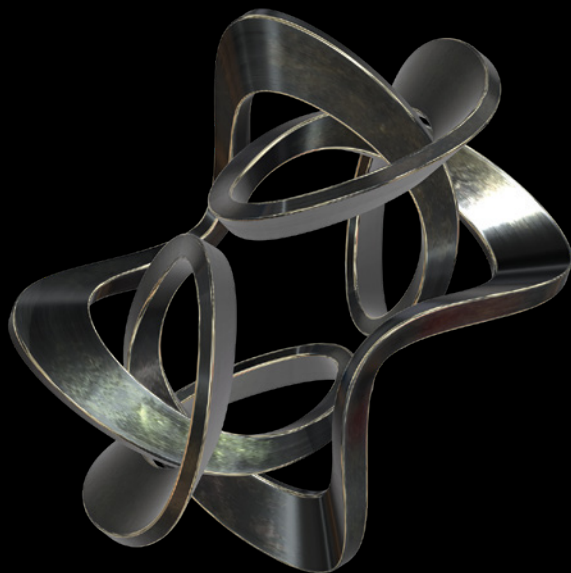
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Perspective 3



Perspective 4



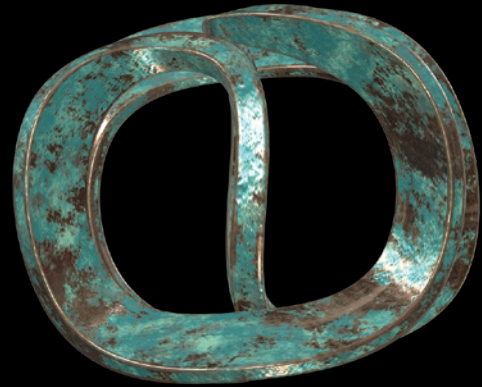
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Perspective 1





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Perspective 3



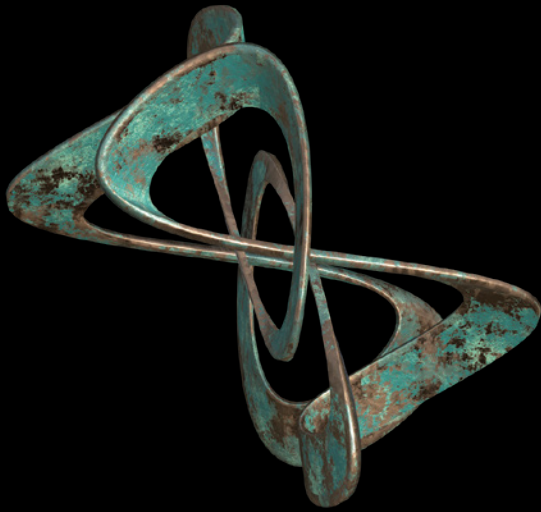
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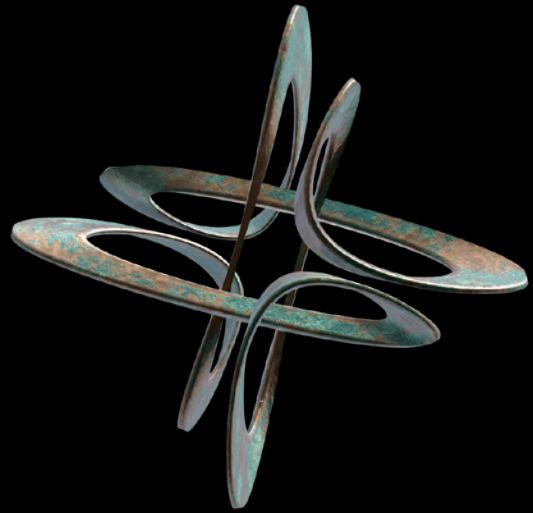
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Perspective 1



Perspective 2



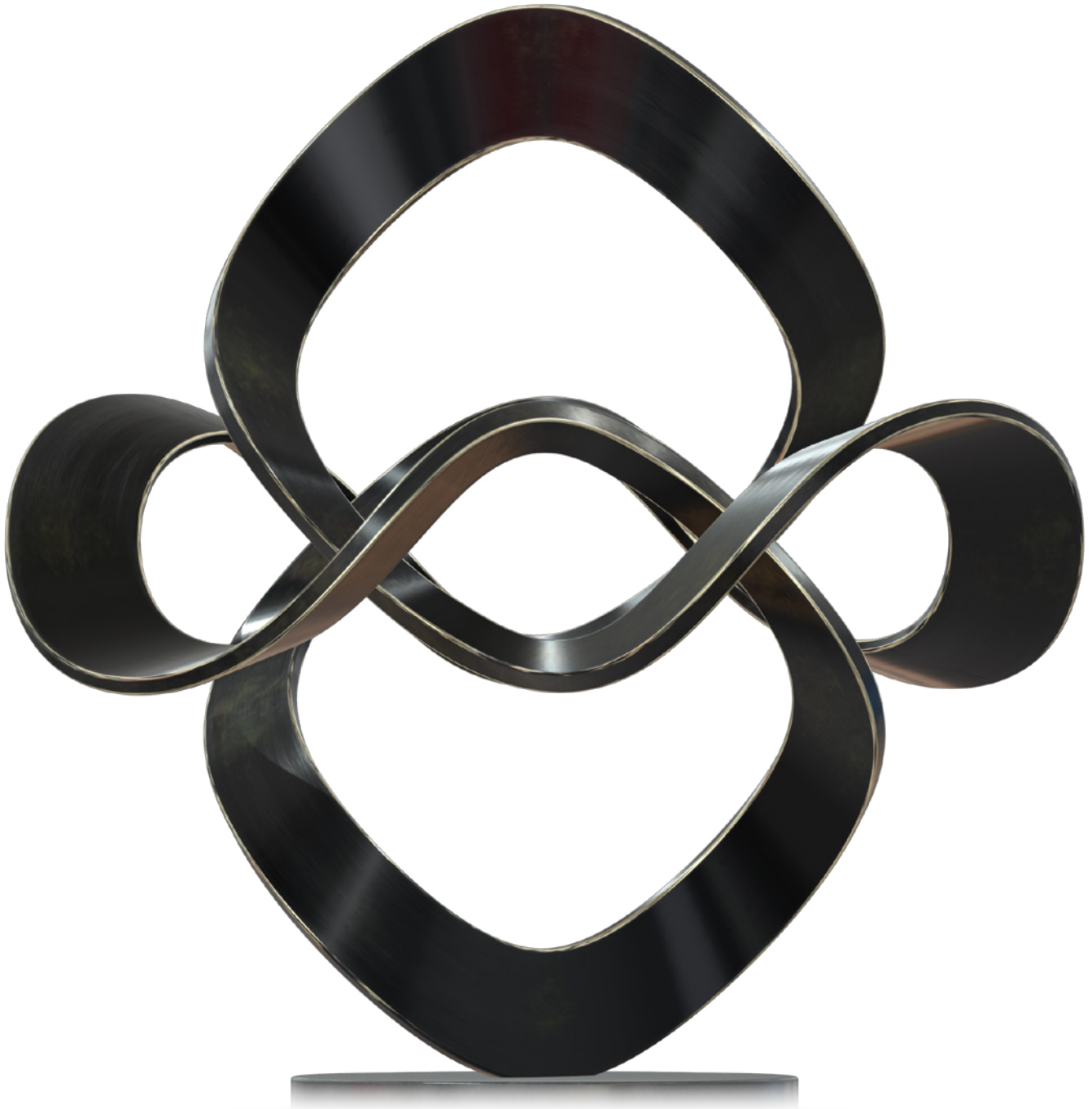
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Perspective 4



Perspective 5



Perspective 1





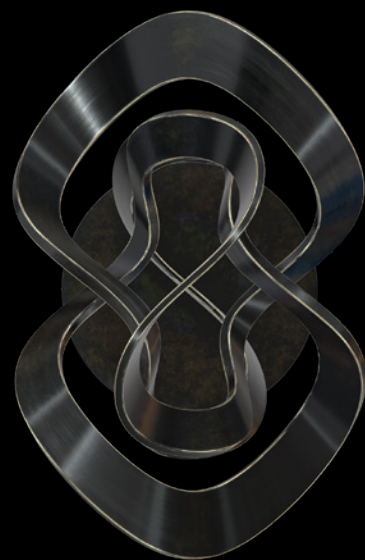
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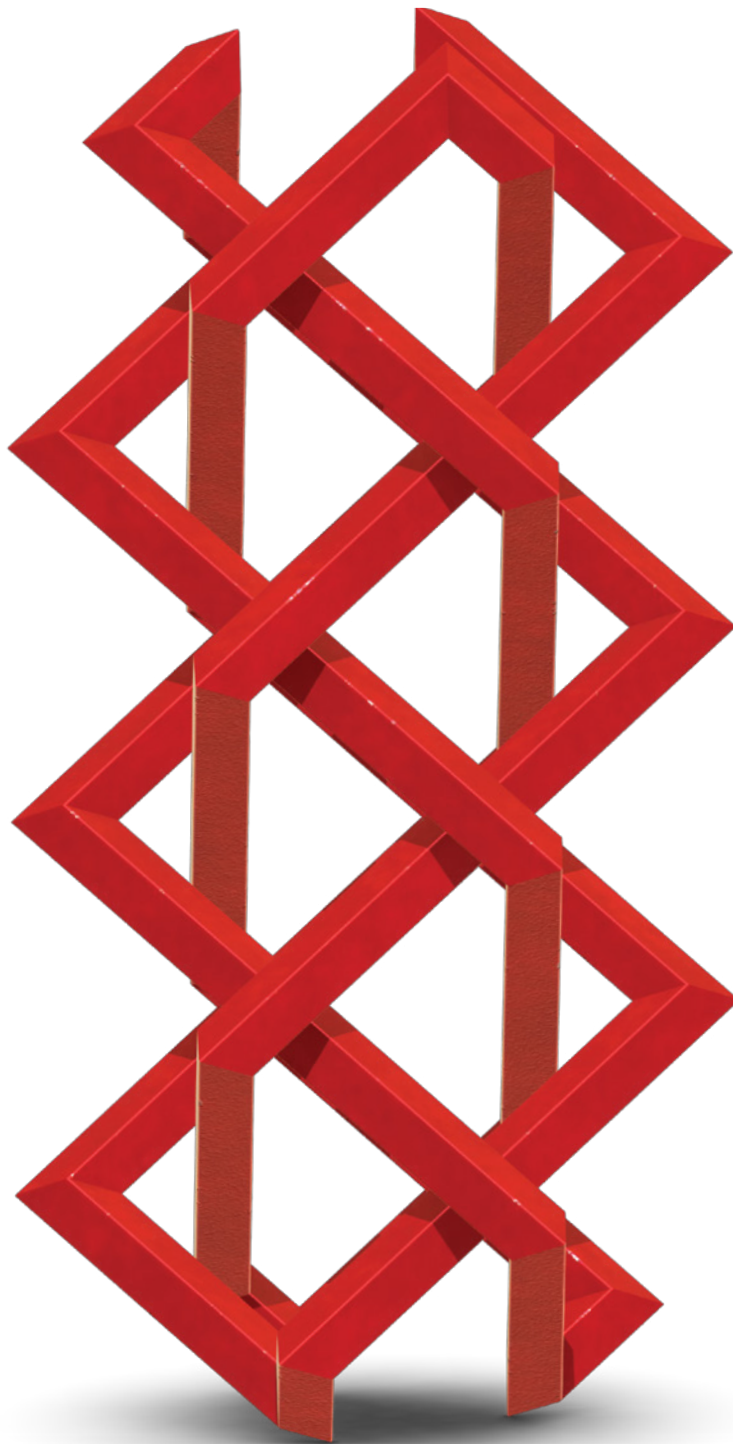


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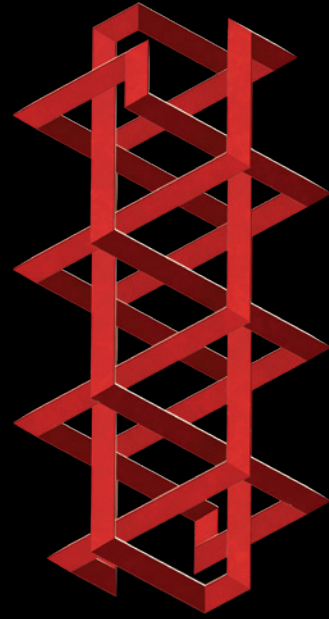


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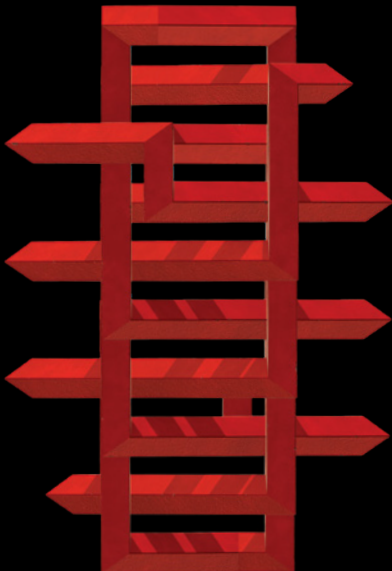
First discovered by Koos in the 1980s



Perspective 2



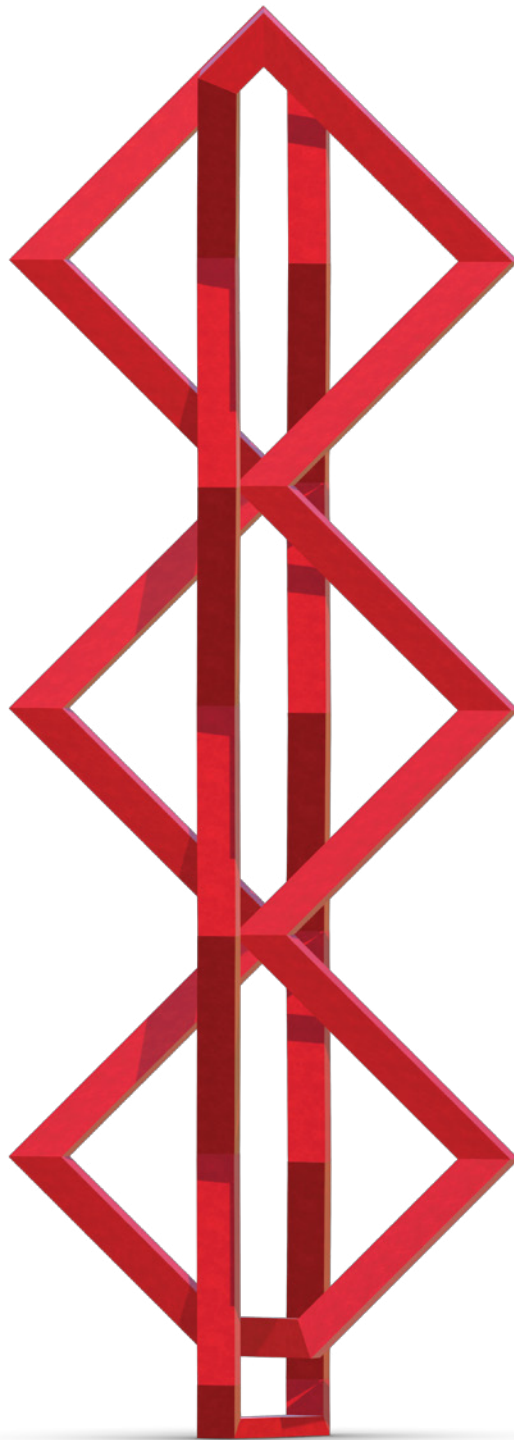
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Perspective 4



Perspective 5

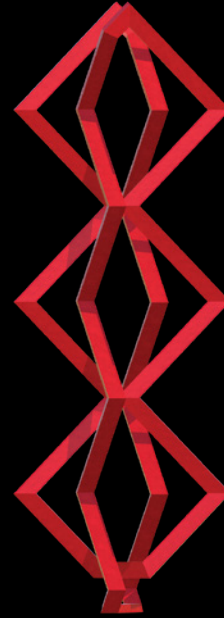


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Perspective 2



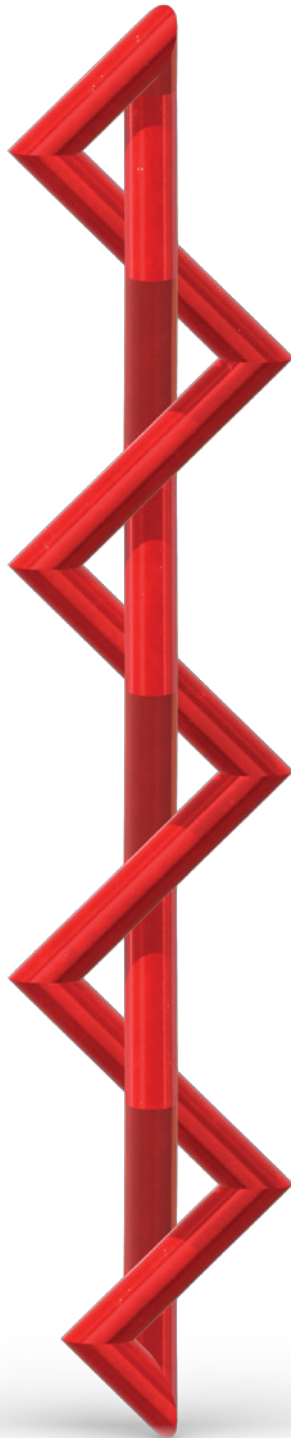
Perspective 3



Perspective 4



Perspective 5



Perspective 1

In-lattice variant of "Hek om niets" by Koos Verhoeff



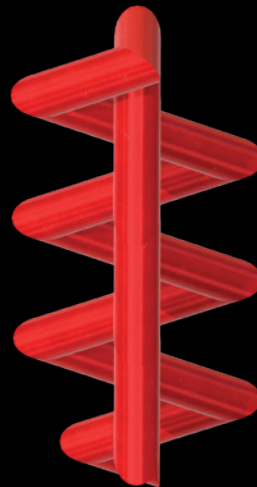
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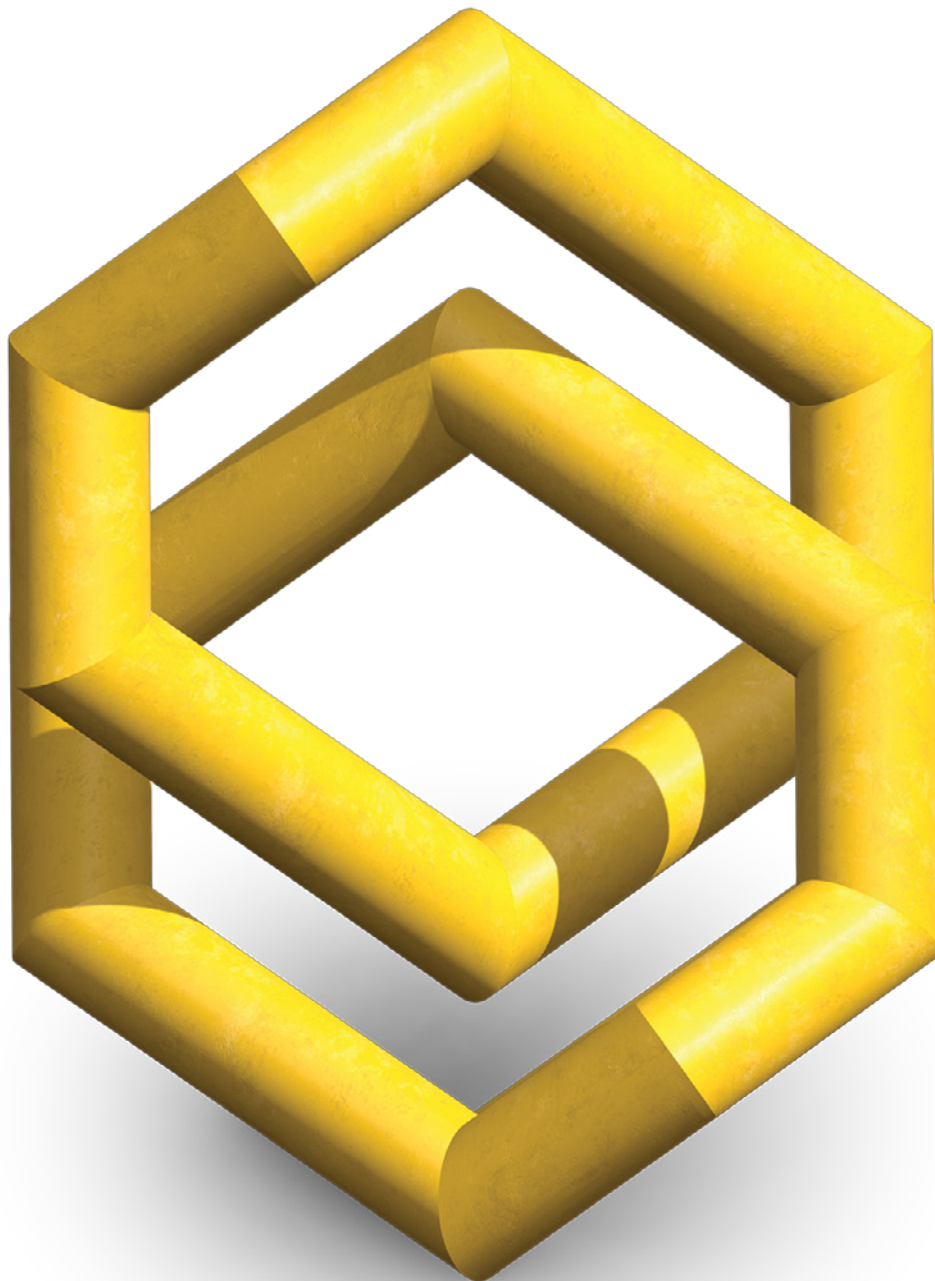
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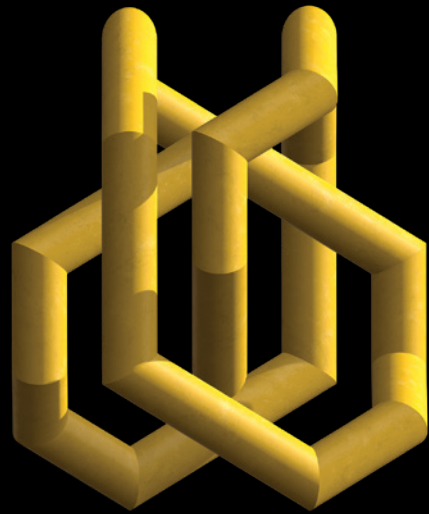
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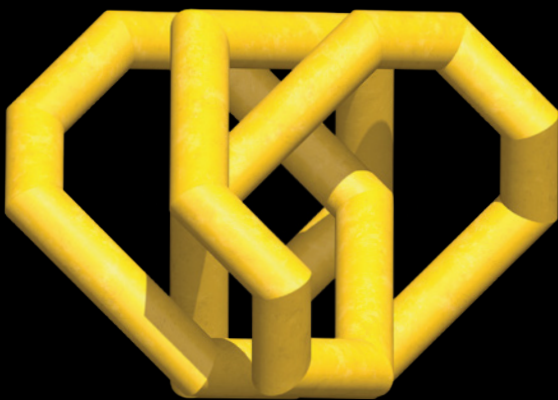
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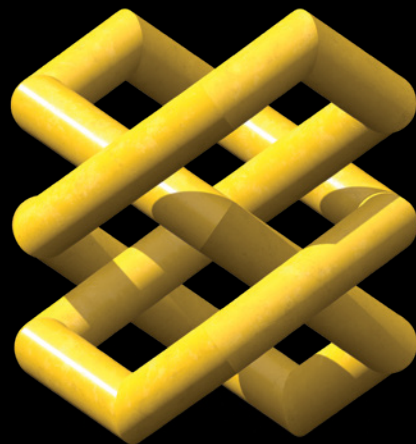
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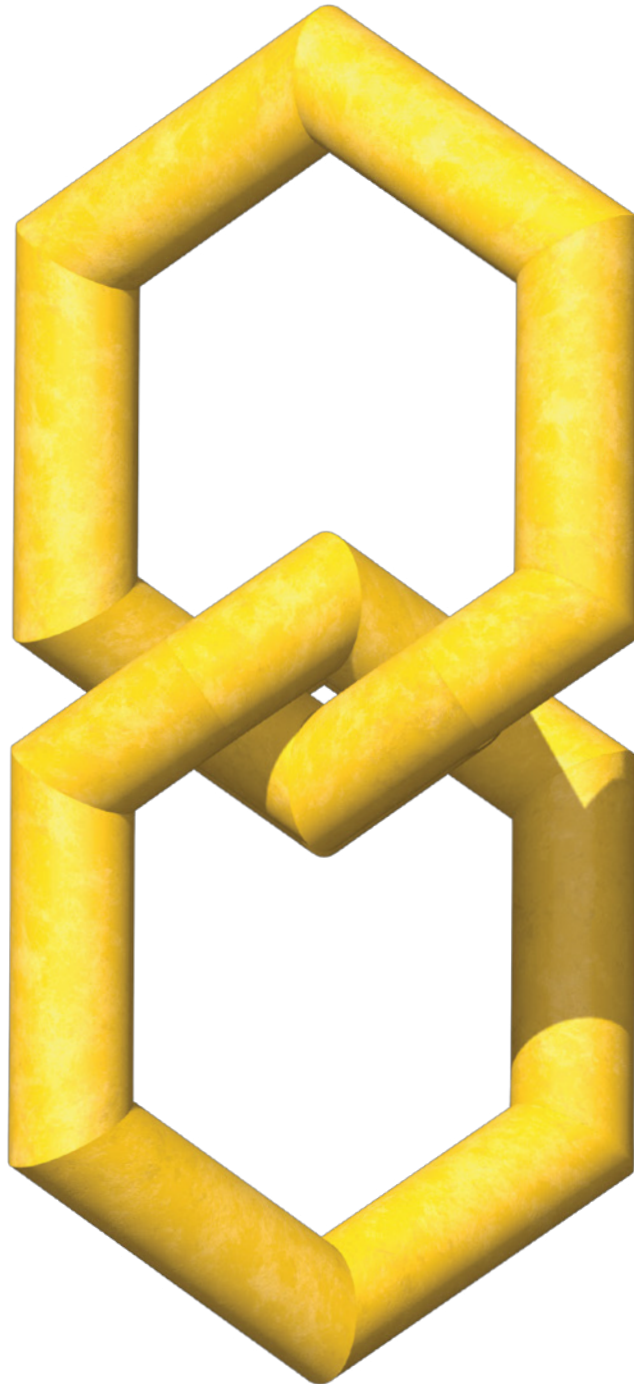
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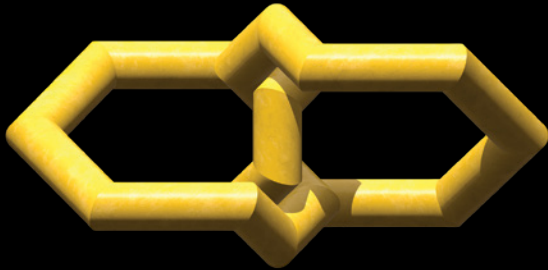
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Perspective 5



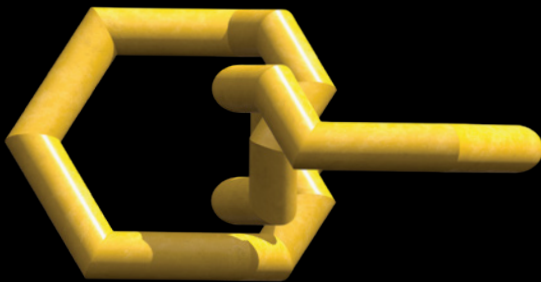
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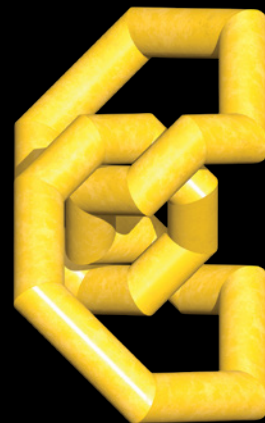
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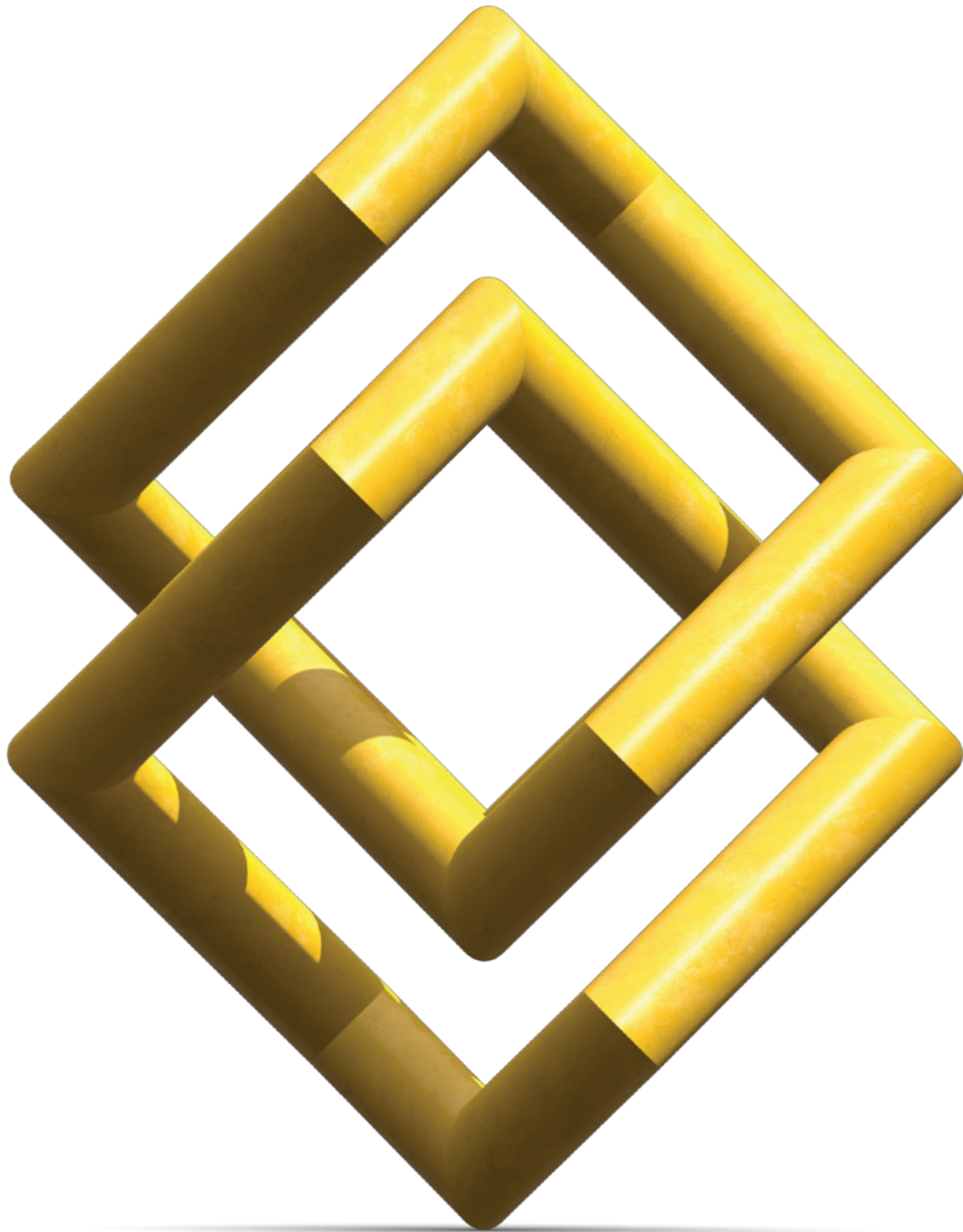


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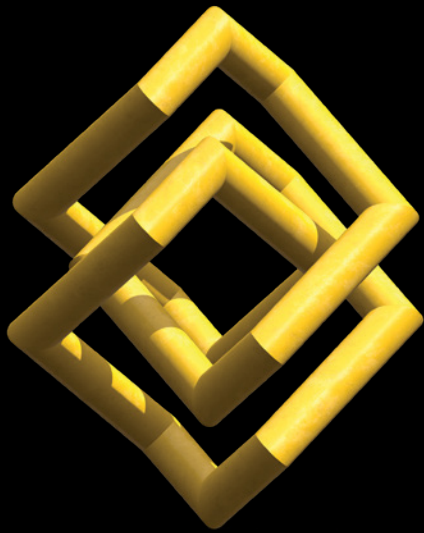
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Perspective 1

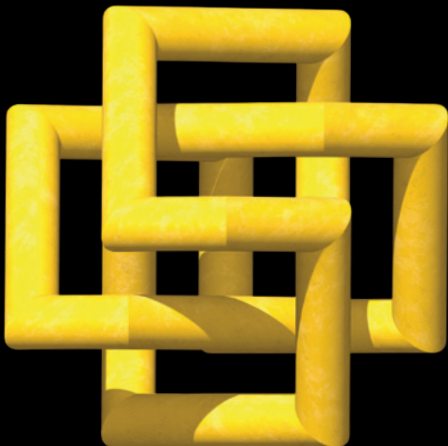




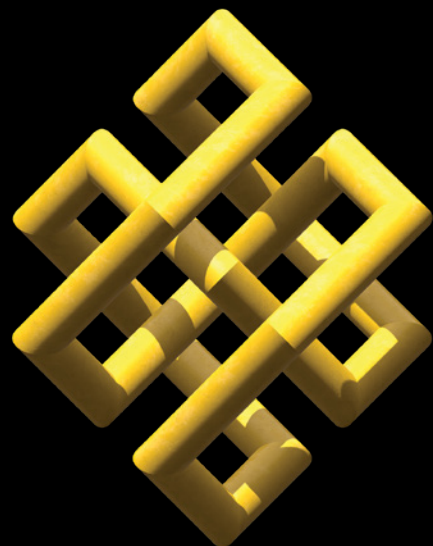
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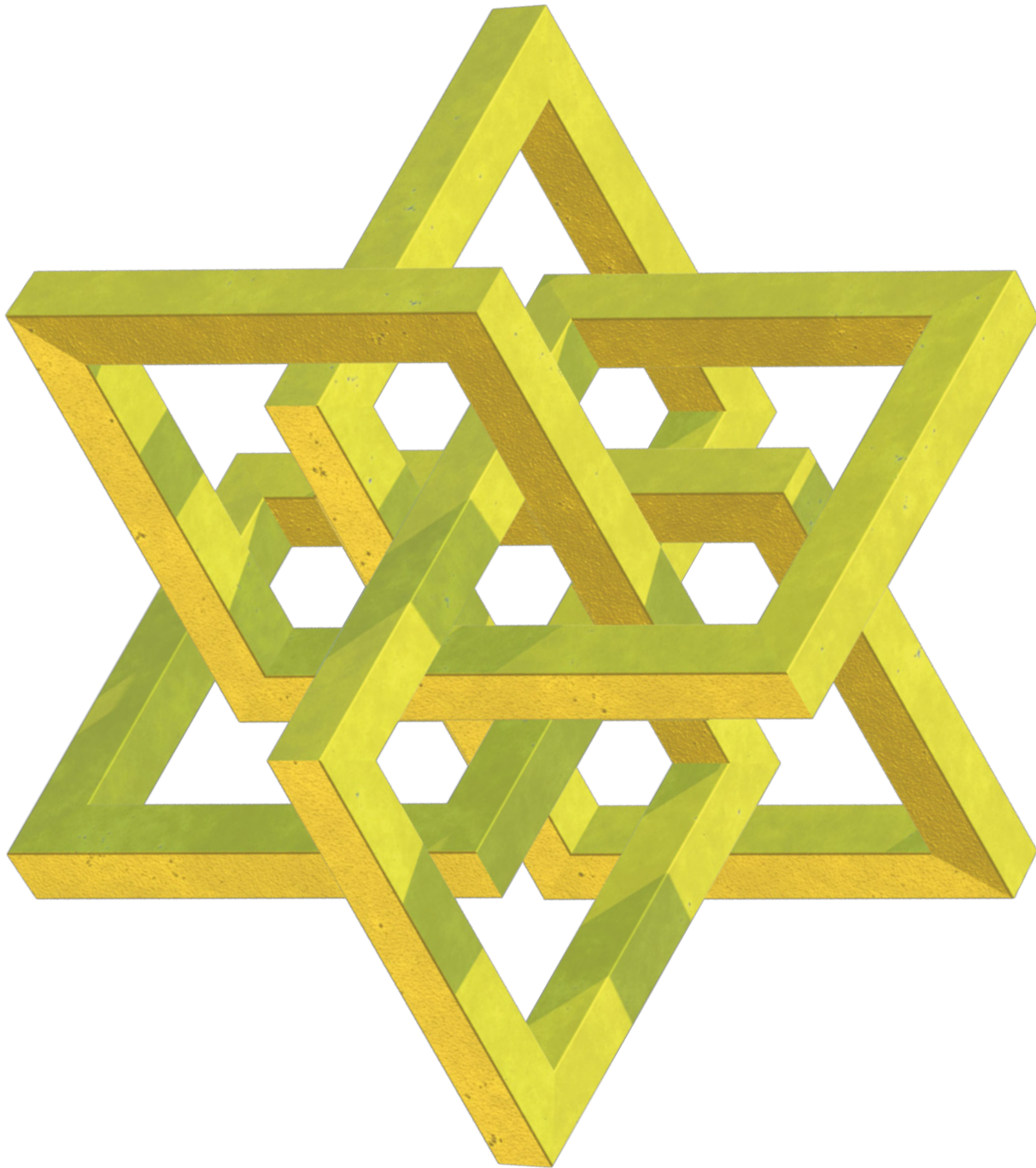
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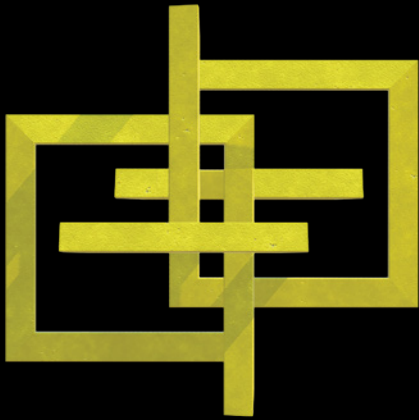
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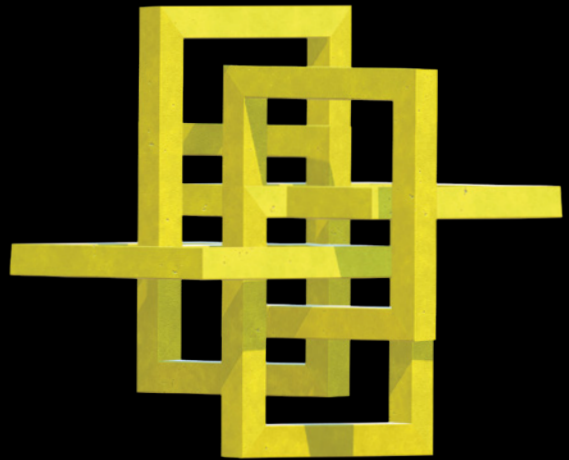
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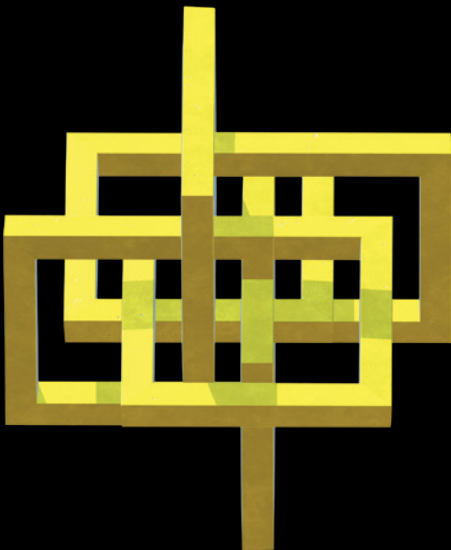
Perspective 1



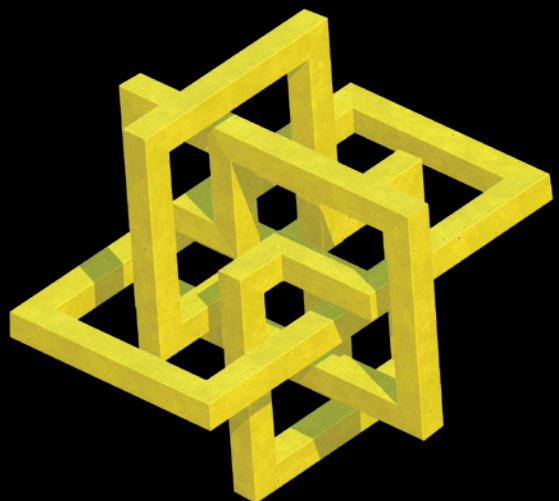
Perspective 2



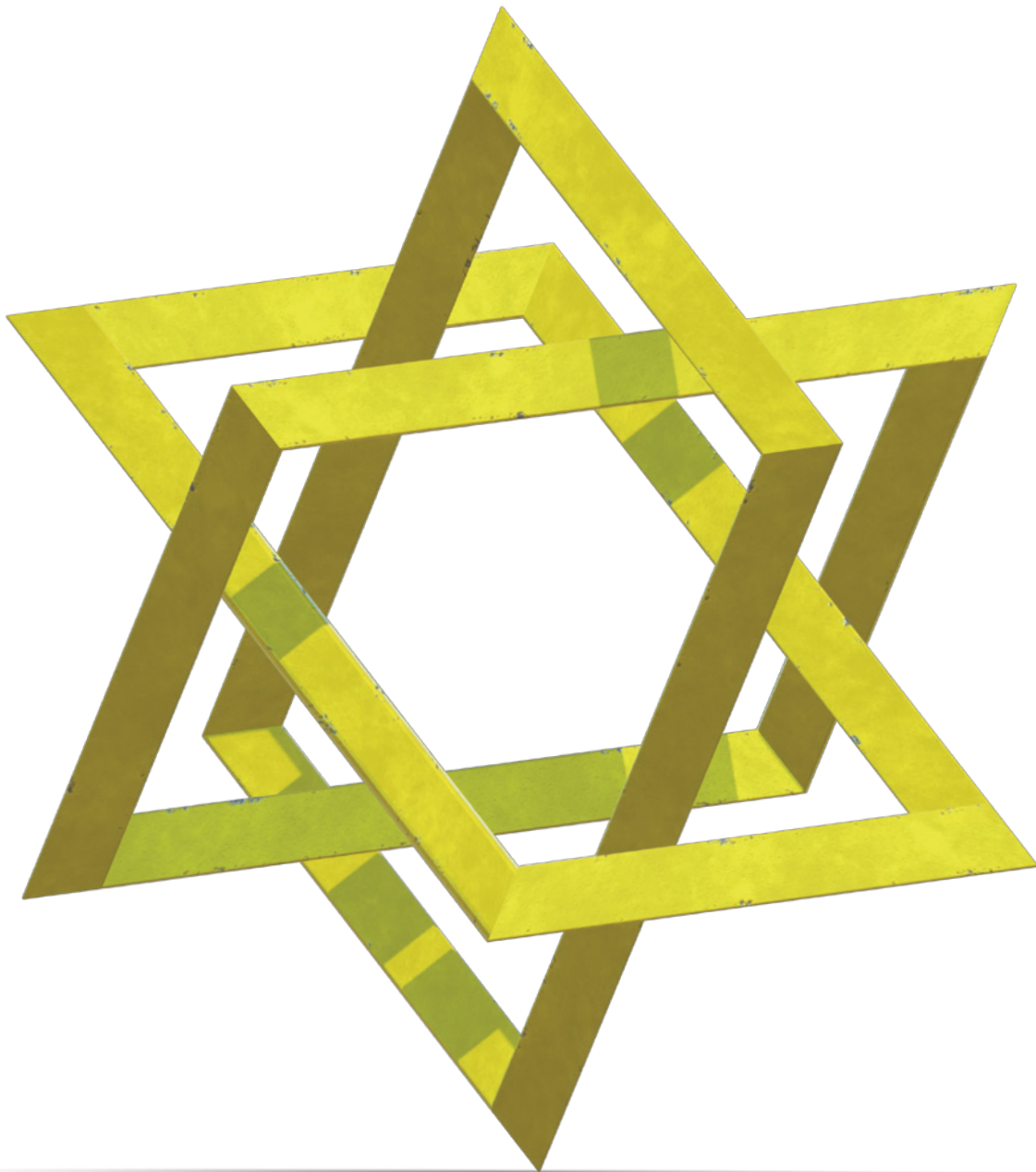
Perspective 3



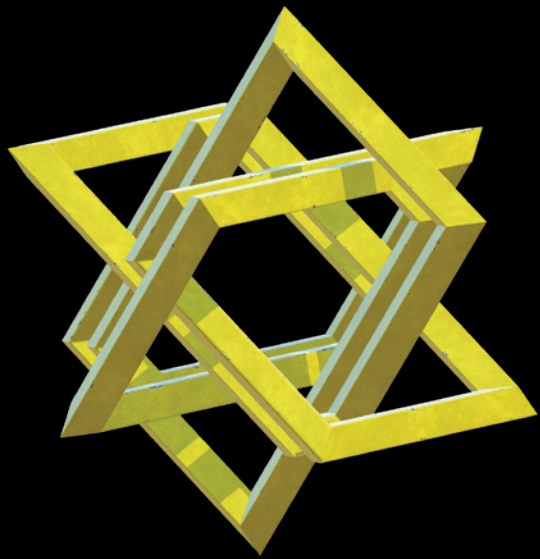
Perspective 4



Perspective 5



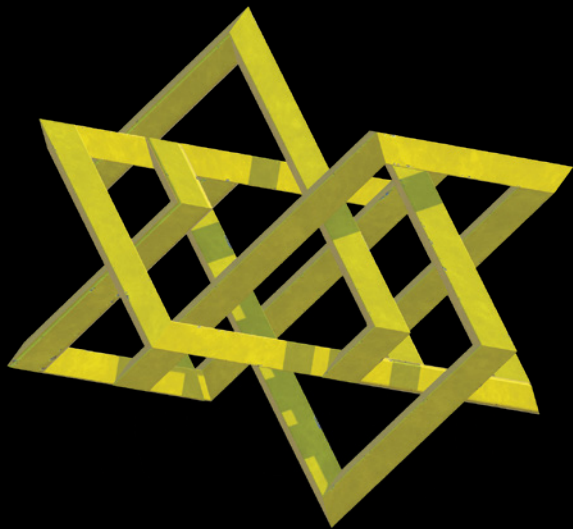
Perspective 1



Perspective 2



Perspective 3

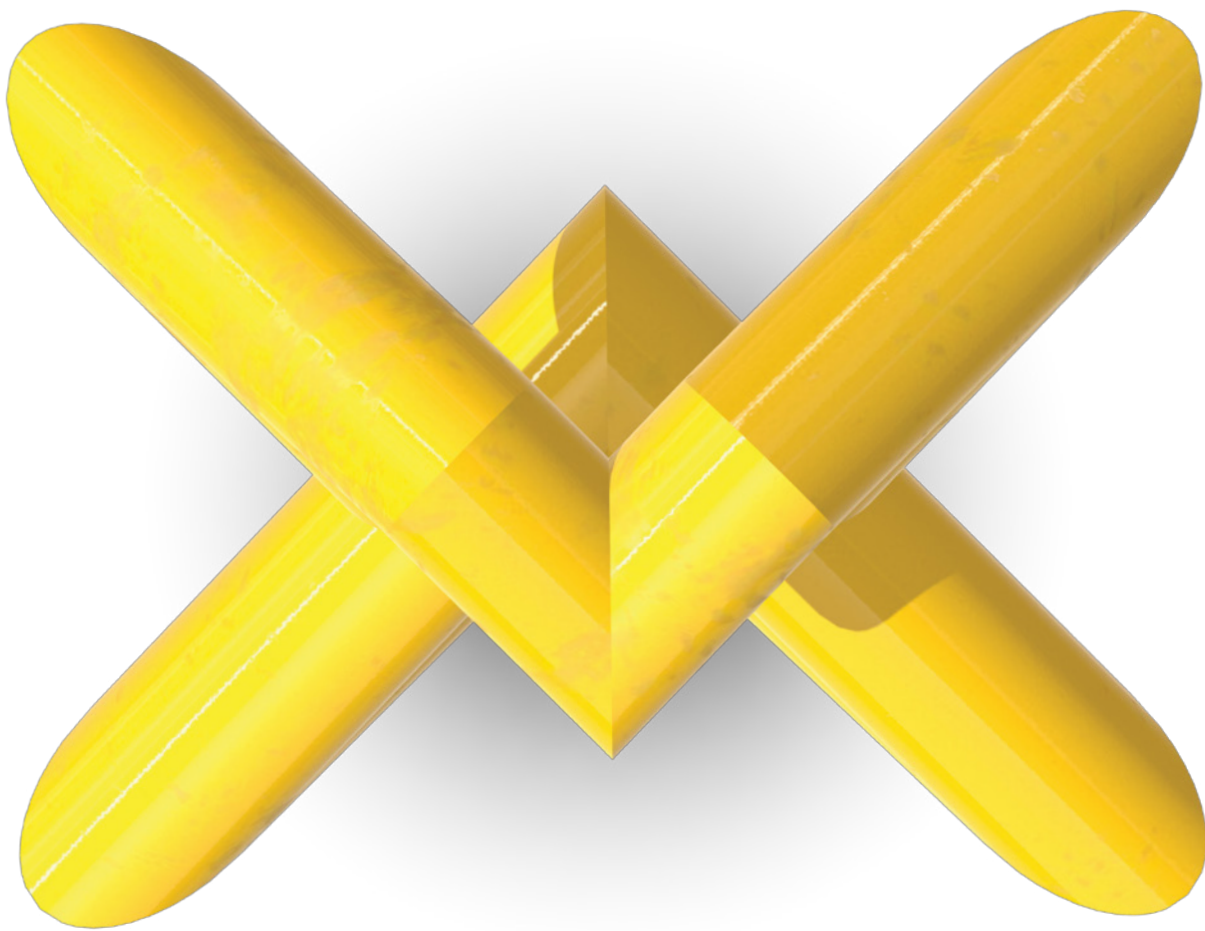


Perspective 4



Perspective 5





Perspective 1





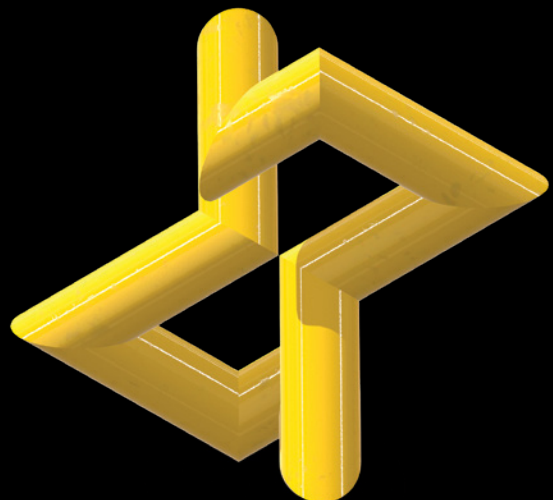
Perspective 2



Perspective 3



Perspective 4



Perspective 5

# Anton Bakker

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